Calculation of Radiation Heat Transfer in a Volume Above the Surface of a Corium Pool

L. A. Dombrovskii
Institute of High Temperatures, Russian Academy of Sciences, ul. Izhorskaya 13/19, Moscow, 127412 Russia

Abstract—The radiation heat transfer in the space above the surface of a pool of corium formed in a severe accident of a nuclear reactor is considered. A group of approximate methods of calculation has been proposed to solve the problem, in particular the solution for a diathermic medium in the transparency windows and the solution in the P_1-approximation or in the modified approximation of radiation conductivity (MRC) for the absorbing and scattering medium. The results of calculations for typical values of the parameters in the problem are given.

If a severe accident can arise in a nuclear power station in which the core of the nuclear reactor is degraded, a pool of molten corium is formed in its vessel, which consists mainly of uranium oxide and also zirconium oxide and molten steel. Nuclear reactions continue in the corium with an intensive generation of heat; as a result of this, the steel vessel of the reactor can be destroyed. In one constructional solution, the corium is further retained in a concrete cavity located under the reactor [1].

In the computational analysis of the thermal interaction between the corium and the reactor vessel, and then with the walls of the cavity, we have to account for the thermal radiation of the corium pool. Estimates of heat transfer show that the role of radiation in both cases is quite substantial and, in the cavity, the thermal radiation from the surface of the pool is the main mechanism for removing heat from the corium, at least until water supply to the cavity begins.

In order to determine the unsteady thermal state of the corium and the walls of the construction, it is necessary to solve the conjugate problem of heat transfer. The calculation of radiation heat transfer is only one of the components of the computational model. Here, the distribution of the density of the integral flux of thermal radiation should be known at every time step, or at least periodically recalculated as the temperature and the composition of the gaseous medium change, and also the temperature of the radiating and reflecting surfaces [2, 3]. Therefore, in order to calculate radiation transfer, we should employ simple approximations.

The purpose of this study is to construct comparatively simple computational models to calculate the transfer of thermal radiation in the volume above the corium pool, which are suitable for solving the general conjugate problem of heat transfer, and also to make a computational analysis of typical distributions of the radiation flux density along the bounding surfaces.

THE LEVEL OF THE MAIN PARAMETERS AND THE TYPICAL OPTICAL PROPERTIES OF THE MEDIUM

Under quasi-steady thermal conditions, the temperature of a large part of the corium pool surface is 2500–3000 K, while the temperature of the open surface of the walls is 1700–1800 K (for the melting steel vessel of the reactor or for the wall of the cavity from chamotte concrete that is being destroyed). The space above the pool can be schematically represented as a cylinder 4–5 m in diameter and 3–5 m in height. The medium in the volume above the corium pool contains steam with a partial pressure from 0.1 to 0.6 MPa and, possibly, a certain amount of aerosols. The latter are most probably present in the concrete cavity. Simple estimates of heat transfer show that thermal radiation is the predominant mechanism of heat transfer from the corium pool to the walls of the reactor or cavity located above it. In all cases, except for the direct supply of water to the volume under consideration, convective heat transfer need not be taken into account; heat transfer should be calculated as in the case of radiative equilibrium [4–7].

The calculation of a purely gaseous medium (without aerosols) is the most complicated. In Fig. 1, we find the spectral relationships of the absorption coefficient Σ of the steam at p = 0.1 MPa for the temperature range of interest; these relationships were calculated using a wide-band model [8]. The recalculation for other pressures is made by multiplying the absorption coefficient by the pressure ratio. The spectral windows occupy a large part of the spectrum; in them, we should use the solution for the diathermic medium [6]. In the bands of absorption, it is necessary to solve the spectral problem of radiative equilibrium. From Fig. 1 we see that the
the long-wave bands, the absorption of steam
partment of
0.6 MPa for a height of the dome of the reactor com-
medium in the short-wave bands is small: even with
order of magnitude higher. The optical thickness of the
section in which the coefficient of absorption is about an
coefficient of absorption and the main bands of absorp-
divided into two groups: short-wave bands with a small
spectral ranges where the optical thickness of the
regions of opacity or great scattering of the radiation
solutions: a certain differential approximation in
approximation of the diathermic medium in the
ranges where the absorption and scattering of the
radiation are small.

The four absorption bands should naturally be sub-
divided into two groups: short-wave bands with a small
coefficient of absorption and the main bands of absorp-
tion in which the coefficient of absorption is about an
order of magnitude higher. The optical thickness of the
medium in the short-wave bands is small: even with
p = 0.6 MPa for a height of the dome of the reactor com-
part ment of H = 3 m, it turns out that \( \tau_0 = \Sigma_a H = 3 \). In
the long-wave bands, the absorption of steam \( \tau_0 \)
changes from 4 to approximately 40, depending on the
partial steam pressure.

The radiation characteristics of the aerosols can be
calculated on the basis of the optical properties of sub-
stances and the distribution of the particles according to
their size, employing Mie theory [4]. The uncertainty of
the chemical composition and the size and concentra-
tion of the particles of the aerosols is very great. How-
ever, with radiative equilibrium, the albedo of the
medium does not influence the solution of the problem,
and it is sufficient to calculate the transport coefficient
of attenuation \( \Sigma_{tr} \), which is not so sensitive to the optical
properties of the substance of the particles [4, 9].
The spectrum of absorption of a gaseous medium with
aerosols is continuous and, with a high concentration of
particles, we can employ the approximation of a gray
medium with coefficients averaged over the spectrum.
In this study, the mean transport coefficient of attenuat-
tion is varied within the following limits: \( 1 < \Sigma_{tr} < 3 \text{ m}^{-1} \).
As will be seen from the analysis that follows, calcula-
tions of radiative equilibrium for \( \Sigma_{tr} > 3 \text{ m}^{-1} \) do not have
any practical meaning because, in this case, it is neces-
sary to cool the surface of the pool with water in order
to prevent the corium from overheating.

**APPORXIMATE METHODS FOR CALCULATING RADIATION TRANSFER**

The distance from the surface of the corium pool to
the dome of the reactor vessel or to the cavity is com-
mensurable with the diameter of the pool. Therefore, in
order to correctly determine the radiation flux density
at the boundary surfaces, we should proceed from the
solution to the two-dimensional problem of radiation
transfer. If the medium in the volume under consid-
eration has a large optical thickness over the entire spec-
trum, the solution can be obtained in some kind of a dif-
ferential approximation, in a \( P_1 \)-approximation, for
example [4, 5]. As was mentioned above, this is valid
for a gaseous medium with a high concentration of
aerosols, which, to a great extent, corresponds to the
conditions in the cavity. For a purely gaseous medium
or with a small amount of aerosols, a formal application
of differential approximations may lead to large errors
in the spectral ranges where the optical thickness of the
medium is small. At the same time, there is hardly any
sense in going over to more complex methods for
describing the radiation transfer. Instead, it seems
advisable to employ a combination of two approximate
solutions: a certain differential approximation in
regions of opacity or great scattering of the radiation
and an approximation of the diathermic medium in the
spectral ranges where the absorption and scattering of
the radiation are small.

To reduce the technical difficulties of the solution,
only a cylindrical volume is considered. The real form
of the dome can be accounted for in particular calcula-
tions by using the same methods.

**THE SOLUTION FOR A DIATHERMIC MEDIUM**

For a diathermic medium, which does not take part
in the transfer of thermal radiation, the problem under
consideration can be formulated as a set of linear alge-
braic equations [6]

\[
\sum_{j=1}^{n} \left( \delta_{kj} \frac{\varepsilon_{kj}}{\varepsilon_{kj}} - F_{k-j} \frac{1}{\varepsilon_{kj}} \right) q_j = q^0_k;
\]

\[
q^0_k = \pi \sum_{j=1}^{n} (\delta_{kj} - F_{k-j}) B_\lambda(T_j),
\]

where \( j \) is the number of the section of the surface on
which the temperature \( T_j \), the emissivity \( \varepsilon_j \), and the
spectral density of the radiation flux \( q_j \) are taken to be
constant; \( \delta_{kj} \) is the Kronecker symbol; \( F_{k-j} \) is the view
factor accounting for the irradiance of section \( j \) from
the side of section \( k \); \( q^0_k \) is the spectral density of the
radiation flux in the \( k \)th section for absolutely black sur-
faces; and \( B_\lambda \) is Plank’s function.

For the model problem under consideration, it is
sufficient to single out the following sections of the sur-
face: the annular elements of the corium pool surface

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Fig. 1. The spectral absorption coefficient for steam at a par-
tial pressure of 0.1 MPa. T, K: (1) 2000; (2) 2500; (3) 3000.
(j = 1, 2, ..., m), the annular elements of the side wall
(j = m + 1, m + 2, ..., n − 1), and the flat dome (j = n).
In this case, we can use the known formulas for the
view factors [6]:
\[ F_{a-j} = f_1(r') - f_1(r_j); \quad F_{j-n} = F_{n-j} - \frac{R^2}{(r'_j)^2 - (r_j)^2}; \]
\[ f_1(r) = 0.5[X - \sqrt{X - 4(R/r)^2}]; \]
\[ X = 1 + [1 + (R/H)^2]l(r/H)^2; \]
\[ F_{k-j} = f_2(r'_j) - f_2(r); \quad F_{j-k} = \frac{2R\Delta z_k}{(r'_j)^2 - (r_j)^2}; \]
\[ f_2(r) = \frac{2R\Delta z_k}{U} - 1; \]
\[ U = 1 + (z_k/R)^2 + (r/R)^2; \]
\[ F_{k-n} = \frac{Y^2 + 0.5}{\sqrt{Y^2 + 1}} - Y; \quad F_{n-k} = \frac{2\Delta z_k}{R}; \]
\[ Y = (H - z_k)/l(2R); \]
\[ F_{k-l} = \left[ 1 - \frac{Z(Z^2 + 1.5)}{(Z + 1)^{1.5}} \right] \frac{\Delta z_k}{2R}; \quad Z = |z_k - z_l|/(2R), \]
where \(1 \leq i, j \leq m; m < k, l < n; r_j\) and \(r'_j\) are the internal
and external radii of annulus \(j\) at the lower base of the
cylinder; \(z_k\) and \(\Delta z_k\) are the mean axial coordinate and
the height of annulus \(k\) on the side surface; \(R\) and \(H\) are
the radius and the height of the volume under consider-
ation.

The set of Eqs. (1) with coefficients (2) are applica-
table to spectral intervals for which the optical thickness
of the medium is small with respect to the absorption
and also to the scattering. With gray surfaces, when the
emissivity is constant over the entire spectrum, we can
immediately determine the integral density of the radia-
tion flux over the region of transparency. It is sufficient
to replace in (1) the expression for the quantity \(q^0_k\) by
the following one
\[ q^0_k = \pi \sum_{j=1}^{n} \left( \delta_{k-j} - F_{k-j} \right) \sum_{l=1}^{m} B_i(T_l) d\lambda. \]

Here, \(\lambda_c < \lambda < \lambda_{c}\) is the spectral interval of transpar-
tency of number \(l\) and \(m\) is the number of such intervals.

### The Solution for an Absorbing
### and Scattering Medium

The solution for an absorbing and scattering medium
can be obtained using the \(P_1\)-approximation, in
which the spectral radiation flux is considered to be
proportional to the gradient of the volumetric density
of the radiation energy of [4, 5]
\[ q_k = -D_k \nabla I^0_k. \] (4)

The corresponding boundary-value problem for a
cylindrical volume has the following form [4]:
\[ -\nabla(D_k \nabla I^0_k) + \Sigma_a I^0_k = \Sigma_a \times 4\pi B_k(T); \] (5)
\[ z = 0, \quad -D_k \frac{\partial I^0_k}{\partial z} = \frac{\gamma_i}{2} [4\pi B_k(T_1) - I^0_k]; \]
\[ z = H, \quad -D_k \frac{\partial I^0_k}{\partial z} = \frac{\gamma_i}{2} [4\pi B_k(T_3) - I^0_k]; \]
\[ r = 0, \quad \frac{\partial I^0_k}{\partial r} = 0; \]
\[ r = R, \quad -D_k \frac{\partial I^0_k}{\partial r} = \frac{\gamma_i}{2} [4\pi B_k(T_2) - I^0_k]. \]

Here, \(D_k = 1/(3\Sigma_a)\) is the spectral coefficient of dif-
fusion of radiation; the index 1 stands for the surface of
the corium pool and the indexes 2 and 3 stand for the
side and the top surfaces of the volume under consider-
ation.

With radiative equilibrium, the temperature field is
found as the solution to the following energy equation:
\[ \nabla q = 0, \quad q = \int q_k d\lambda \] (6)
or
\[ \nabla \left( \int D_k \nabla I^0_k d\lambda \right) = 0. \] (7)

### The Model of a Gray Medium for a High
### Concentration of Aerosols

For a high concentration of aerosols, we can use the
model for a gray medium with radiation characteristics
that are average over the spectrum. In this case, the
problem is simplified, in principle, and takes the fol-
lowing form:
\[ \nabla (D \nabla \vartheta) = 0, \quad \vartheta = 4\sigma T^4; \] (8)
The calculation of the radiation heat transfer, as follows:

\[ \bar{q} = \frac{q}{\sigma(T_1^4 - T_2^4)} = \frac{1}{3\tau_0/4 + 1/\gamma}; \quad \tau_0 = \Sigma \varepsilon H. \quad (12) \]

It is not difficult to show that, in the RC-approximation, \( \bar{q} = 4/(3\tau_0) \), and, in the MRC-approximation,

\[ \bar{q} = \frac{\chi}{\ln[(1 + \chi/\gamma)\exp(\sqrt[3]{\alpha}/2) - \chi/\gamma]}; \quad \chi = 2\sqrt[3]{\alpha/3}; \quad \alpha = \Sigma \varepsilon / \Sigma \sigma. \quad (13) \]

From the table we see that the MRC-approximation is much more accurate than the usual approximation of radiative conductivity. For a large optical thickness \( (\tau_0 \gg 1) \), formula (13) takes the form

\[ \bar{q} = \frac{1}{3\tau_0/4 + \ln(1 + \chi/\gamma)/\chi}; \quad \chi = 2\sqrt[3]{\alpha/3}; \quad \alpha = \Sigma \varepsilon / \Sigma \sigma. \quad (14) \]

The transition to the two-dimensional problem of radiation transfer greatly increases possible errors in \( P_1 \), and especially those in the MRC-approximation. An analysis of the accuracy of the \( P_1 \)-approximation made in [4, 11–13] shows that, in the problem under consideration, the largest error should be in the solution in the angle zone where the corium pool comes into contact with the side wall. Here, \( P_1 \) may overestimate the radiation flux to the wall. In this zone, we should expect the largest error from the MRC-approximation as well, because the optical thickness of the medium that subdivides the surface of the pool and the adjacent side wall is very small. Actually the temperature of the pool surface should coincide with the wall temperature. Therefore, it is not necessary to try to achieve a formally strict solution for the density of the radiation flux in the angle zone; the error will be the less, the greater the role of thermal radiation in the adjacent region.

The Solution for Absorption Bands

The spectral calculation of radiative equilibrium in the absorption bands shown in Fig. 1 can be carried out
The absorptions of the pairs of short-wave and long-wave bands are taken to be the same, and the absorption factor is assumed to be constant for the short-wave bands and variable for the long waves. The formal consideration of the approximation of the long-wave bands is not significant. As a result, we should assess the influence of the long-wave bands on the temperature profile, using the solution for the radiative equilibrium in these absorption bands. As a result, instead of (15) and (16), it is sufficient to solve two boundary-value problems similar to (8) and (9), in which \( \theta \) takes part instead of the function \( S_j(T) \), where \( j = 1, 2 \) is the number of the group of the bands.

A much simpler approximate solution is obtained if we consider that the long-wave absorption bands make up a small share of the integral flux of thermal radiation. First, we can solve the problem of the radiative equilibrium for the short-wave bands, assuming that the change in the temperature profile as a result of radiation in the long-wave bands is not significant. After this, we should assess the influence of the long-wave bands on the temperature profile, using the solution for the radiative equilibrium of these absorption bands. As a result, instead of (15) and (16), it is sufficient to solve two boundary-value problems similar to (8) and (9), in which \( \theta \) takes part instead of the function \( S_j(T) \), where \( j = 1, 2 \) is the number of the group of the bands. A numerical solution of these problems was obtained using the method proposed in [14].

**RESULTS OF CALCULATIONS**

The data from the calculations presented below are for a cylindrical concrete cavity with a diameter of 2\( R = 5 \) m; the height of the dome above the surface of the corium pool is \( H = 3 \) m. The emissivity of all the surfaces is taken to be the same and equal to \( \varepsilon_0 = 0.8 \). The temperatures of the side wall of the cavity and the dome are assumed to be the same and equal to \( T_w = 1800 \) K; the maximal temperature of the pool surface is \( T_s = 2700 \) K.

**RADIATION HEAT TRANSFER IN THE TRANSPARENCY WINDOWS**

The results of a calculation of the integral radiation flux in the transparency windows for steam for the above-mentioned parameters of the problem are given in Fig. 2, where the coordinate \( s \) of the development of the boundary of the computational domain is plotted along the abscissa. As previously, a positive value of \( \tilde{q}_i \) on the pool surface means that heat is removed, and positive values of \( q_i \) and \( q_s \) on the walls means that heat is supplied to the wall. The profile of the temperature of the pool surface was given by the formula

\[
T_s = T_c - (T_c - T_w) \left( \frac{r - r_0}{R - r_0} \right)^2 \theta(r - r_0), \tag{18}
\]

where \( \theta \) is the Heaviside function. Alternatives with \( r_0 = R \) (an isothermal surface) and \( r_0 = 0.8R \) were con-
sidered. In the non-isothermal alternative, the density of the radiation flux to the side wall of the cavity is maximum at a distance equal to about the thickness of the thermal boundary layer in the corium (in the alternative under consideration, it is 0.5 m). It is interesting that the reflection of radiation from the wall leads to a situation in which the radiation flux near the wall may be directed to the pool surface (for negative values of $q_1$ in Fig. 2).

Note that the integral radiation flux in the transparency windows at a temperature of the pool surface of 2700 K is large enough to remove the greater part of the heat generated in the volume of the corium pool. However, in this case, the maximal density of the heat flux to the wall (300–400 kW/m²) is dangerous to the concrete wall of the cavity.

RADIANT HEAT TRANSFER IN THE ABSORPTION BANDS

The results of a calculation of the density of the integral radiation flux in the absorption bands for steam are presented in Fig. 3. The proposed approximate solution with a separate calculation of the radiative equilibrium in the short-wave and the long-wave bands was used. In Fig. 3, we also find profiles of the density of the radiation flux that were calculated for the temperature field, which is a solution to the following boundary-value problem:

$$\Delta T = 0; \quad z = 0, \quad T = T_1; \quad z = H, \quad T = T_3;$$
$$r = 0, \quad \frac{\partial T}{\partial r} = 0; \quad r = R, \quad T = T_2. \quad (19)$$

We see that the influence of the temperature profile on the radiation flux is quite insignificant, and the error of the approximate solution for the density of the radiation flux that is integral along the absorption bands will most likely not exceed 10%.

A comparison of the data in Figs. 3 and 2 for the transparency windows attests to the considerable role of irradiation in the short-wave bands and the insignificant contribution of the long-wave absorption bands. As with spectral windows, the maximal density of the radiation flux to the cavity wall is observed in the lower section of the side surface; in the absorption bands, however, this effect is more pronounced. Note that the observed increase in the density of the radiation flux at the periphery of the pool surface, which is stronger with a high partial pressure of the steam, should, together with the conductivity, lead to a lower temperature of the corium in this zone.

RADIATION TRANSFER IN A GASEOUS MEDIUM WITH AEROSOLS

In Fig. 4 we find calculated profiles of the density of the integral radiation flux for radiative equilibrium in a medium containing aerosols. It is assumed that the aerosol particles are spherical with a radius of $a = 1 \mu$m, and the complex index of refraction of the substance of the particles $m = 1.5 - 0.1i$. The transport factor of the efficiency of attenuation $Q_t$ was calculated using Mie theory [4]. The corresponding attenuation factor is

$$\Sigma_u = 0.75 f_v \frac{Q_t}{a}, \quad (20)$$

where $f_v$ is the volumetric concentration of the aerosols. Two alternatives of radiative equilibrium in the $P_1$-approximation were calculated: for an isothermal surface of the pool with $T_s = 2700$ K and for a variable temperature $T_s$ given by formula (18), in which $T_{sw} = 2400$ K is substituted for $T_{sw} = 1800$ K. We see that, on the iso-
thermal surface of the pool, very high densities of the radiation flux are observed in the zone where the corium adjoins the side wall. The relatively small decrease in the temperature of the pool surface near the wall greatly reduces the maximal density of the radiation flux to the wall; this maximum, however, still lies next to the pool surface. An increase in the aerosol concentration reduces the radiation flux from the central part of the pool surface and the flux going to the remote walls.

As was shown above, we can employ the MRC-approximation when the optical thickness of the medium is large. At the same time, it would not be correct to formally assign discontinuous boundary conditions (for example, with a jump in the temperature at the boundary between the corium pool and the wall), because this would lead to an infinite local density of the heat flux. In the conjugate problem of heat transfer (in which the temperature field in the corium is found simultaneously), this does not arise. Assigning the temperature profile (18) does not create such difficulties. The following boundary-value problem was solved:

\[ \nabla(MDV\vartheta) = 0; \quad z = 0, \quad \vartheta = \vartheta_1; \]
\[ z = H, \quad \vartheta = \vartheta_3; \quad r = 0, \quad \frac{\partial \vartheta}{\partial r} = 0; \]
\[ r = R, \quad \vartheta = \vartheta_2. \]

The density of the radiation flux was found from formula (11). In calculating the coefficient \( M \), it was assumed that \( S_0 = S_0' \). The results of the calculation shown in Fig. 5 attest to the satisfactory accuracy of the MRC-approximation, which may be used for solving the conjugate problem of heat transfer in the computational domain, which includes a pool of stratified corium and the volume above the pool surface containing aerosols.

\[ q, \text{kW/m}^2 \]

Fig. 5. The density of the integral flux of thermal radiation for the temperature profile of the pool surface in (18) and the volumetric concentration of the aerosols \( f_v = 3 \times 10^{-6} \).

CONCLUSION

(1) To calculate the radiation heat transfer in a steam-containing volume above the corium pool, we employed the solution for a diathermic medium in the transparency windows together with a calculation of the radiative equilibrium in the absorption bands using the \( P_1 \)-approximation. A simple approximate method for solving the spectral problem of the radiative equilibrium was proposed for these conditions. The additional error in finding the density of the integral radiation flux in the absorption bands associated with the above is not greater than 10%.

(2) When a great amount of aerosols is present, we can use the model for a gray medium. In this case, we can employ either the \( P_1 \)-approximation or (in the case of an optically dense medium) the modified approximation of radiative conductivity (MRC) for calculating the radiative equilibrium.

(3) Our calculations of the radiation heat transfer for typical parameters of the problem of cooling the corium in the concrete cavity show that, for a purely gaseous medium, the transparency windows for the steam make up more than a half of the radiation flux. In this case, the side wall of the cavity near the pool surface is subjected to an especially strong impact of thermal radiation. These specifics of the distribution of the radiation flux are retained in the presence of aerosols, when the heat flux removed from the central part of the pool surface is much smaller.

(4) The results of calculations confirm the possibility of employing the MRC-approximation for simulating radiation transfer in solving the conjugate problem of the thermal state of the corium pool when the volumetric concentration of the aerosols is \( f_v \geq 3 \times 10^{-6} \).

REFERENCES


